

BOSON MAPPINGS OF THE FERMION DYNAMICAL SYMMETRY MODEL AND STRUCTURAL EVOLUTION OF $^{186-192}\text{Os}$

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Abstract. This work examines the microscopic connection between the fermion dynamical symmetry model (FDSM) and an effective bosonic description of collective nuclear states. A simplified FDSM Hamiltonian containing monopole and quadrupole pairing terms together with proton and neutron multipole interactions is mapped onto an s - d boson space by three procedures: the generalized Dyson mapping, a seniority-based Hermitization, and the Belyaev–Zelevinsky expansion. The role of the physical boson subspace, Hermiticity, and the truncation of the boson expansion to one- and two-body operators is discussed. The Dyson representation preserves the fermionic matrix elements in the appropriate physical subspace but is generally non-Hermitian in the conventional boson metric. The seniority and Belyaev–Zelevinsky procedures provide Hermitian IBM-like Hamiltonians that can be compared directly with the interacting boson model. The mapped Hamiltonians are applied to the even osmium isotopes ^{186}Os , ^{188}Os , ^{190}Os , and ^{192}Os . The effective parameters evolve comparatively smoothly from ^{186}Os to ^{190}Os , whereas several coefficients show a pronounced rearrangement at ^{192}Os . The same mass region is singled out by different mappings, supporting an interpretation in terms of a change in the effective collective structure. The parameter discontinuity alone is not taken as proof of a quantum phase transition; further tests using spectra, $B(E2)$ values, quadrupole observables, and collective energy surfaces are required.

Keywords: Fermion dynamical symmetry model; interacting boson model; boson mapping; Dyson mapping; seniority; Belyaev–Zelevinsky mapping; collective nuclear structure; osmium isotopes.

1. INTRODUCTION

Collective excitations in medium-mass and heavy nuclei provide a characteristic example of emergent behavior in a finite quantum many-body system. Low-lying states of even-even nuclei often display vibrational, rotational, or transitional regularities although the underlying degrees of freedom are interacting protons and neutrons. A direct shell-model description becomes increasingly difficult as the valence space grows, and this has motivated collective truncation schemes in which the dominant correlated degrees of freedom are treated explicitly [1, 2, 3, 4, 5, 6, 7]

The interacting boson model (IBM) is one of the most successful algebraic descriptions of such collective motion. Its basic degrees of freedom are s and d bosons with angular momenta $L = 0$ and $L = 2$. The dynamical limits of the model reproduce broad classes of collective spectra, while the Otsuka–Arima–Iachello construction provides a microscopic interpretation of the bosons as correlated nucleon pairs [1, 2, 3, 4, 5]. This microscopic connection is important because it allows the parameters of an effective boson Hamiltonian to be related, at least in principle, to the structure of a fermionic valence space.

Pairing and seniority provide a natural bridge between shell-model and boson descriptions. When low-lying states are dominated by coherent pairs, a large fermion space can be reduced to a much smaller subspace containing a limited number of broken pairs. The generalized seniority concept extends this idea to multi-orbit configurations and has remained useful for understanding collective truncations with realistic interactions [6, 7].

The fermion dynamical symmetry model (FDSM) develops a related idea directly in fermion space. Collective S and D pairs are embedded in a dynamical algebra, so that collective modes arise from the symmetry properties of the underlying nucleonic system rather than being introduced as elementary bosons [8, 9, 10, 11]. The FDSM therefore offers a useful framework for asking a precise microscopic question: under what conditions can a fermionic collective Hamiltonian be represented by an effective IBM-like boson Hamiltonian?

Boson mappings address this question by constructing a correspondence between selected fermion states and operators and their images in a boson space. The mapping is not unique. Different procedures distribute the effects of the Pauli principle, finite fermion-space dimensionality, and normalization among the boson operators, the metric, and the definition of the physical subspace [12, 13, 14, 15, 16, 17, 18]. The generalized Dyson mapping is algebraically efficient and can preserve fermionic matrix elements in the physical sector, but its nonunitary nature leads in general to a non-Hermitian boson image. Seniority-based similarity transformations and Belyaev–Zelevinsky (BZ) expansions provide alternative Hermitian representations suitable for direct comparison with the conventional IBM [13, 14, 15, 16, 17, 18].

A particularly relevant study is the work of Dobeš, Navrátil, and Geyer, who investigated boson mappings of the FDSM and showed how a seniority-dictated Hermitization of the Dyson image and a BZ mapping truncated to one- and two-body terms lead to IBM-type Hamiltonians [15]. The present paper follows this physical line but focuses on the parameter systematics for even osmium isotopes and on the structural information contained in the mapped interactions.

The osmium chain is an appropriate testing ground because nuclei in this region display transitional collective behavior. IBM analyses of Pt–Os nuclei and detailed studies of ^{192}Os have long indicated a significant evolution of collective structure in this mass region [19, 20]. Modern microscopic IBM approaches based on energy-surface mapping likewise emphasize the sensitivity of collective Hamiltonians to deformation and pairing degrees of freedom [21, 23].

The aim of this work is therefore twofold. First, we formulate the Dyson, seniority, and BZ mappings of a simplified FDSM Hamiltonian in a common notation and clarify the approximations required to obtain Hermitian IBM-like operators. Second, we analyze the mapped Hamiltonian parameters for ^{186}Os , ^{188}Os , ^{190}Os , and ^{192}Os and identify the robust trends shared by different mappings. The original numerical parameter sets are retained, while the physical interpretation is formulated more cautiously: a rearrangement of effective collective interactions is distinguished from a demonstrated quantum phase transition.

2. THEORETICAL MODEL

In the approximation considered here, the collective FDSM space is generated by correlated S and D fermion pairs. The S pair carries total angular momentum $J = 0$ and represents monopole pairing, whereas the D pair carries $J = 2$ and describes quadrupole pair correlations. The restriction to these collective pairs is consistent with the general microscopic picture that low-angular-momentum pairs dominate a large part of the low-energy structure of many even-even nuclei [5, 22].

We neglect broken-pair configurations and pair scattering between normal- and abnormal-parity levels. This assumption defines the model space used for the mapping; it should not be interpreted as a statement that such configurations are absent in the physical nucleus. Within this simplified space the model admits $\text{Sp}(6) \otimes \text{SO}(2)$ and $\text{SO}(8) \otimes \text{SO}(2)$ dynamical symmetry structures, depending on the active coupling scheme [8, 9, 15].

The general proton-neutron two-body Hamiltonian in this limit contains eleven parameters. In the

original formulation used for the present application it is written as follows:

$$H_{\text{FDSM}} = \sum_{k,i} \varepsilon_{ki} n_{ki} + \sum_{a,a'} G_0^{aa'} S^\dagger(a) S(a') + G_2 D^\dagger \cdot D + \sum_{r,a,a'} B_r^{aa'} P^r(a) \cdot P^r(a') \quad (1)$$

Here ε_k and n_k are the single-particle energy and the number operator of level k (the index k labels the fermion type); a and a' label the proton and neutron subsystems; $S^\dagger(a)$ and $D^\dagger(a)$ create the collective S and D pairs of subsystem a ; $P^r(a)$ is the multipole operator of rank r ; and the dot denotes the scalar product of two tensors of equal rank.

For applications to the osmium isotopes the Hamiltonian is further reduced by retaining the dominant pairing and quadrupole terms. The resulting five-parameter form is

$$H = G_{0\pi} S_\pi^\dagger S_\pi + G_{0\nu} S_\nu^\dagger S_\nu + B_{2\pi} P_\pi^2 \cdot P_\pi^2 + B_{2\nu} P_\nu^2 \cdot P_\nu^2 + B_{2\pi\nu} P_\pi^2 \cdot P_\nu^2 \quad (2)$$

Here π and ν denote proton and neutron degrees of freedom. The first two terms represent monopole pairing in the two subsystems, while the remaining terms describe quadrupole interactions within and between the proton and neutron sectors. This reduced form preserves the competition between pairing and quadrupole collectivity that is central to the structural evolution studied below [8, 22].

2.1. Electromagnetic Quadrupole Operator

The electromagnetic quadrupole operator is taken in the one-body form

$$T(E_2) = l_\pi P_\pi^2 + l_\nu P_\nu^2 \quad (3)$$

Mapping observables is as important as mapping the Hamiltonian. Agreement in energy eigenvalues alone does not guarantee that the mapped wave functions retain the transition properties of the original fermion problem. A complete validation should therefore include $B(E2)$ values and static quadrupole moments in addition to level energies [3, 4, 5, 16]. The source calculation reports spectra and electromagnetic transition probabilities; the present paper concentrates on the mapping structure and on the parameter systematics available in the original data set. A detailed numerical calculation of the excitation spectra, $B(E2)$ values, and static quadrupole moments with the mapped Hamiltonians is reserved for future work and lies outside the scope of the present analysis of parameter evolution.

3. BOSON MAPPING OF THE FDSM OPERATORS

A boson mapping is a correspondence between a selected fermion subspace and a boson space together with a mapping of the operators acting in those spaces. The construction is subtle because the fermion space is finite and constrained by the Pauli principle, whereas the ideal boson Fock space is infinite. Consequently, the mapped boson space can contain spurious states unless a physical subspace is identified [12, 13, 14, 15, 16].

The practical objective is to reproduce fermionic matrix elements while obtaining an operator form simple enough for numerical work and comparison with the IBM. Exactness, Hermiticity, and low operator rank cannot always be achieved simultaneously. The three mappings considered below represent different compromises among these requirements.

3.1. Generalized Dyson Mapping

The FDSM Hamiltonian can be mapped onto a boson space by the generalized Dyson transformation. For the $\text{Sp}(6)$ and $\text{SO}(8)$ fermion algebras the images are expressed through s - and d -boson creation and annihilation operators. In the notation of the original model, the monopole-pair, quadrupole, dipole, and octupole operators are mapped as

$$S^\dagger \rightarrow \sqrt{\Omega} \left(s^\dagger - \frac{1}{\Omega} s^\dagger s^\dagger s - \frac{1}{\Omega} s^\dagger (d^\dagger d)^{(0)} - \frac{1}{\Omega} (d^\dagger d^\dagger)^{(0)} s - \frac{1}{\Omega} \chi ((d^\dagger d^\dagger)^{(2)} d)^{(0)} \right) \quad (4)$$

$$S \rightarrow \sqrt{\Omega} s \quad (5)$$

$$P^2 \rightarrow (d^\dagger s + s^\dagger d)^{(2)} + \chi (d^\dagger d)^{(2)} \quad (6)$$

$$P^1 \rightarrow \sqrt{2} (d^\dagger d)^{(1)}, \quad P^3 \rightarrow \sqrt{2} (d^\dagger d)^{(3)} \quad (\text{SO}(8) \text{ case}) \quad (7)$$

$$P^4 \rightarrow \frac{1}{\sqrt{2}} \sqrt{15} (d^\dagger d)^{(4)} \quad (\text{Sp}(6) \text{ case}) \quad (8)$$

In these expressions Ω denotes the fermion pair degeneracy and χ is the algebra-dependent quadrupole coefficient. The notation $(AB)^{(\lambda)}$ denotes the tensor product of the operators A and B coupled to total angular momentum λ ; in particular, $(d^\dagger d)^{(0)}$ and $(d^\dagger d^\dagger)^{(0)}$ are the scalar couplings, $(d^\dagger d)^{(2)}$ is the quadrupole coupling, and $((d^\dagger d^\dagger)^{(2)} d)^{(0)}$ couples three d -boson operators to a scalar [cite: 6]. The normalization and phase conventions of these couplings are those of Ref. [15]. In the same notation $n_s = s^\dagger s$ is the s -boson number operator. The equations have been retained from the original manuscript because their normalization and phase conventions are part of the model definition.

To diagonalize the Dyson-mapped Hamiltonian, the physical boson basis must be selected consistently. The mapping is constructed so that the physical matrix elements agree with those obtained in the fermion space. However, because the Dyson transformation is nonunitary, the image of a Hermitian fermion Hamiltonian is generally non-Hermitian with respect to the standard boson scalar product [12, 13, 14, 15].

This feature distinguishes the Dyson image from the conventional Hermitian IBM Hamiltonian. It does not imply that the mapping is physically incorrect: within the physical subspace the fermionic information can be preserved. Nevertheless, for direct comparison with an IBM parametrization it is desirable to construct a Hermitian equivalent or approximation. This motivates the seniority and BZ procedures discussed next.

3.2. Seniority Mapping and Hermitization

In a seniority-based mapping, fermion states with well-defined seniority ν are related directly to boson states with a fixed number of d bosons. The simplest correspondences are

$$|N, \nu = 0\rangle \leftrightarrow |n_s = N\rangle \quad (9)$$

$$|N, \nu = 2\rangle \leftrightarrow |n_s = N - 1, n_d = 1\rangle \quad (10)$$

The purpose of this construction is to remove the mixture of different d -boson numbers that can occur in a Dyson image of a state with fixed fermion seniority. The images of the S -pair creation and annihilation operators are constrained by

$$S^\dagger \rightarrow \sqrt{\Omega} \left(s^\dagger - \frac{1}{\Omega} s^\dagger s^\dagger s - \frac{2}{\Omega} s^\dagger (d^\dagger d)^{(0)} \right) \quad (11)$$

$$S \rightarrow \sqrt{\Omega} s \quad (12)$$

For the $SU(2)$ pairing subalgebra these expressions provide a Hermitian boson image of the fermion pairing Hamiltonian. Other mapped operators can then be obtained by imposing the appropriate commutation relations or by a similarity transformation from their Dyson images. In general such transformations can generate infinite series, but the lowest orders are sufficient for the IBM-like approximation considered here [13, 15].

For the quadrupole operator the seniority image is

$$P_c^2 = (s^\dagger d)^{(2)} + \left(1 - \frac{n_s}{\Omega + 1 - 2N + 2n_s}\right) (d^\dagger s)^{(2)} + \chi \left(1 - \frac{2n_s}{\Omega - 2N + 2n_s}\right) (d^\dagger d)^{(2)} \quad (13)$$

The two-body terms in Eq. (13) contain the s -boson number operator [cite: 6]. The total number of fermion pairs, and therefore the total boson number N , is fixed. To reduce the quadrupole image to an effective one-body form, two approximations are considered. In seniority mapping A, the s -boson number operator is replaced by its value for the $\nu = 2$ state. In seniority mapping B, it is replaced by an expression involving the average seniority $\langle \nu \rangle$ of the principal FDSM states.

The average seniority $\langle \nu \rangle$ of the principal FDSM states is defined through the expectation value of the pair operator $S^\dagger S$:

$$\langle S^\dagger S \rangle = \frac{1}{4} (2N - \langle \nu \rangle) (2\Omega - 2N - \langle \nu \rangle + 2) \quad (14)$$

With these two prescriptions, the Hermitian quadrupole images become

$$P_{C,A}^2 = \sqrt{1 - \frac{N-1}{\Omega-1}} (d^\dagger s + s^\dagger d)^{(2)} + \chi \left(1 - \frac{2N-2}{\Omega-2}\right) (d^\dagger d)^{(2)} \quad (15)$$

$$P_{C,B}^2 = \sqrt{1 - \frac{N - \frac{1}{2}\langle \nu \rangle - 1}{\Omega - 1 - \langle \nu \rangle}} (d^\dagger s + s^\dagger d)^{(2)} + \chi \left(1 - \frac{2N - \langle \nu \rangle - 2}{\Omega - \langle \nu \rangle - 2}\right) (d^\dagger d)^{(2)} \quad (16)$$

The structure of Eq. (15) is closely related to the Otsuka–Arima–Iachello mapping, while Eq. (16) is closer in spirit to the extension in which seniority information is incorporated more explicitly [3, 4, 5, 6, 24]. These similarities support the interpretation of the seniority procedure as a microscopic bridge between the FDSM collective pair space and the standard s – d boson representation.

The seniority mapping is nevertheless an approximation once the operator dependence on boson number is replaced by a state-specific or averaged value. Its reliability must ultimately be judged by observables rather than by formal Hermiticity alone.

3.3. Belyaev–Zelevinsky Mapping

In the Belyaev–Zelevinsky method the boson images of the multipole operators have the same form as in the Dyson mapping, whereas the pair operators are constructed so as to satisfy the required commutation algebra while preserving Hermiticity [cite: 6]. The method originated as a microscopic boson expansion for anharmonic collective motion [18, 25].

In general, the image of a fermion pair operator is an infinite series in boson operator. For the present purpose we seek an IBM-like Hamiltonian containing only one- and two-body boson terms, so the BZ expansion is truncated consistently at that order. For the $SO(8)$ pairing operator the retained form is

$$S^\dagger \rightarrow s^\dagger \sqrt{\Omega - 2N} + \frac{1}{2}(s^\dagger s^\dagger - (d^\dagger d^\dagger)^{(0)})_s \frac{\sqrt{\Omega + 4} - \sqrt{\Omega - 2N}}{N + 2} \quad (17)$$

In the SO(6) limit this expression reproduces the relevant low-lying matrix elements specified by the FDSM mapping. For Sp(6) symmetry the corresponding expression is

$$S^\dagger \rightarrow s^\dagger \sqrt{\Omega - 3N} - \left[(d^\dagger d^\dagger)^{(0)}_s - s^\dagger n_d - 2s^\dagger s^\dagger s + \frac{\sqrt{7}}{2} \left(d^\dagger (d^\dagger d)^{(2)} \right)^{(0)} \right] \frac{\sqrt{\Omega + \frac{3}{2}} - \sqrt{\Omega - 3N}}{3N + \frac{3}{2}} \quad (18)$$

Combining these expressions with their Hermitian conjugates and retaining only one- and two-body contributions yields a Hermitian boson image of the fermion pairing interaction. Thus both the seniority and BZ procedures produce practical IBM-type Hamiltonians, although the coefficients generated by the two mappings need not be identical.

Comparative studies of boson mappings have shown that OAI, BZ-Marshalek, and related expansion methods can reproduce similar low-energy results when the collective subspace is well developed, but they can differ as Pauli corrections and higher orders become more important [16]. This is precisely why comparing several mappings is useful: trends that survive the change of mapping are more likely to reflect the underlying collective structure rather than a specific parametrization.

4. EFFECTIVE BOSON HAMILTONIANS AND PARAMETER COMPARISON

After mapping, the FDSM Hamiltonian is expressed in terms of s and d bosons and can be organized into a one-body d -boson energy and two-boson interactions coupled to different angular momenta. The detailed coefficients depend on the mapping, the pair degeneracy, the boson number, and the normalization of the fermion operators. Therefore, absolute parameter values from different representations should not be compared term by term unless the corresponding operators have first been brought to a common convention.

In the original calculation the parameters were converted to a common system so that the four coefficients a_1 – a_4 of the Belyaev–Zelevinsky and seniority-A Hamiltonians correspond to analogous terms of the full SU(6) boson Hamiltonian. The numerical values are retained below exactly as in the source tables.

For both the Belyaev–Zelevinsky and the seniority-A mappings the effective one- and two-body boson Hamiltonian is written in the common form

$$H_B = a_1 \hat{O}_1 + a_2 \hat{O}_2 + a_3 \hat{O}_3 + a_4 \hat{O}_4 \quad (18)$$

where a_i ($i = 1$ – 4) are the coefficients listed in Tables 2 and 3 and $\hat{O}_i$ are the corresponding boson operators.

Table 1. Parameters of the SU(6) boson Hamiltonian for osmium isotopes (MeV).

A	ε	C_0	C_2	C_4	ϑ_0	ϑ_2
186	−0.25	0.20	−0.10	0.04	−0.31	0.04
188	−0.89	0	−0.03	0.11	−0.37	0.03
190	−0.23	−0.08	−0.07	0.09	−0.27	0.03
192	−1.98	0.78	0.16	0.41	0.60	0.20

Table 2. Parameters of the Belyaev–Zelevinsky Hamiltonian for osmium isotopes (MeV). The coefficients a_1 – a_4 multiply the boson operators $\hat{O}_1$ – $\hat{O}_4$ defined in Eq. (19).

A	a_1	a_2	a_3	a_4
186	2.01	0.70	0.25	0.02
188	2.10	0.74	0.27	0.04
190	2.15	0.80	0.32	0.05
192	1.30	0.39	0.10	0.18

Table 3. Parameters of the seniority-A Hamiltonian for osmium isotopes (MeV). The coefficients a_1 – a_4 multiply the boson operators $\hat{O}_1$ – $\hat{O}_4$ defined in Eq. (19).

A	a_1	a_2	a_3	a_4
186	2.27	1.02	0.52	0.12
188	2.35	1.05	0.58	0.14
190	2.54	1.12	0.59	0.18
192	0.65	1.14	0.62	0.21

5. RESULTS

Belyaev–Zelevinsky parameter evolution. For ^{186}Os – ^{190}Os , the first three BZ parameters change gradually: a_1 increases from 2.01 to 2.15 MeV, a_2 from 0.70 to 0.80 MeV, and a_3 from 0.25 to 0.32 MeV. The fourth parameter also increases, but remains small, from 0.02 to 0.05 MeV. This behavior is consistent with a progressive modification of the effective collective interaction as neutrons are added.

At ^{192}Os the pattern changes. The first three parameters decrease to 1.30, 0.39, and 0.10 MeV, respectively, whereas a_4 increases to 0.18 MeV. The simultaneous rearrangement of several coefficients is more informative than a fluctuation in a single parameter because it indicates a change in the balance among different components of the effective Hamiltonian.

Seniority-A Parameter Evolution. In the seniority-A representation, a_1 rises from 2.27 MeV in ^{186}Os to 2.54 MeV in ^{190}Os and then drops sharply to 0.65 MeV in ^{192}Os . By contrast, a_2 and a_3 continue a smoother increase, and a_4 changes from 0.12 to 0.21 MeV across the chain. Thus the first coefficient is especially sensitive to the transition from $A = 190$ to $A = 192$ in this mapping.

The BZ and seniority-A procedures do not yield the same numerical coefficients, as expected from their different construction. Nevertheless, both identify ^{192}Os as the isotope at which the preceding smooth trend is interrupted. This mapping-independent feature is the central numerical observation of the present analysis.

Comparison with the SU(6) Boson Parametrization. The SU(6) parameter set also changes strongly at $A = 192$. The one-boson parameter ε reaches -1.98 MeV, C_0 changes to 0.78 MeV, C_4 rises to 0.41 MeV, and v_2 changes sign and becomes 0.60 MeV. Because the SU(6), BZ, and seniority-A Hamiltonians are not identical parametrizations, a direct equality of coefficients is neither expected nor required. The relevant observation is that the isotope at which the trends reorganize is common to the three descriptions.

The numerical evidence therefore supports a structural rearrangement of the effective collective interaction near ^{192}Os . The source manuscript associated this behavior with a change of nuclear shape. In the present revision we retain the observation but avoid treating the parameter discontinuity by itself as proof of a specific prolate-to-oblate quantum phase transition.

6. DISCUSSION

The Pt–Os region has long been recognized as a transitional area in algebraic collective models. Bijker, Dieperink, Scholten, and Spanhoff described the Pt and Os isotopes within the IBM and demonstrated systematic changes of the collective Hamiltonian across the region [19]. A detailed experimental study of ^{196}Os by Casten and collaborators interpreted its level structure as the transition between the $O(6)$ limit and a symmetric rotor [20]. These independent studies provide a physical context for the parameter rearrangement found here.

The FDSM offers a complementary interpretation because the collective pairs remain explicitly fermionic before mapping. Consequently, the Pauli principle, pair degeneracy, and valence-particle number enter the construction before the effective boson Hamiltonian is obtained. The $Sp(6)$ FDSM has itself been shown to exhibit changes of intrinsic geometry as a function of valence-particle number, including axial-to-triaxial evolution caused by finite-space and Pauli effects [9]. This makes the structural sensitivity of mapped parameters physically plausible.

Nevertheless, several distinct concepts must be separated. Gamma softness, stable triaxiality, prolate-oblate evolution, and a quantum phase transition are not synonymous. A discontinuity or rapid variation in fitted or mapped Hamiltonian parameters can signal a change of effective dynamics, but classification of the nuclear shape requires observables that are directly sensitive to the wave functions.

For the present isotope chain, the next essential test is a uniform calculation of the level schemes and electromagnetic properties with the same mapped Hamiltonians. Useful indicators include $R_{4/2} = E(4_1^+)/E(2_1^+)$, the excitation energies of 0_2^+ and 2_2^+ states, odd-even staggering in the gamma band, $B(E2)$ branching ratios, and static quadrupole moments. The original study states that spectra and electromagnetic transition probabilities were obtained, but the available source tables used in this revision contain only the Hamiltonian parameters. Therefore, no new numerical $B(E2)$ values are introduced here. The detailed numerical calculation of these observables is reserved for future work; the present study is deliberately limited to the analysis of the evolution of the mapped Hamiltonian parameters.

A second issue is the truncation of the BZ expansion. Higher-order boson terms can encode Pauli corrections and the finite dimension of the underlying fermion space [13, 15, 16, 25]. If these terms become more important in a transitional nucleus, a one- and two-body truncation may require additional renormalization. The agreement of the location of the parameter rearrangement in two different Hermitian mappings is encouraging, but a quantitative uncertainty analysis would strengthen the conclusion.

A third issue concerns the microscopic interpretation of effective IBM parameters. Modern approaches map self-consistent mean-field or density-functional energy surfaces onto an IBM Hamiltonian [21, 23]. These energy-surface mappings differ formally from the operator mappings used here, but both approaches seek to connect bosonic collective dynamics with nucleonic structure. The density-functional framework has also been

extended to include pairing vibrations, emphasizing that shape and pairing collective coordinates can be coupled [23]. Such developments suggest a useful future comparison with the FDSM, where pairing is present explicitly in the starting Hamiltonian.

The principal result of this work is therefore deliberately stated at the level supported by the available calculations: the effective boson parameters obtained from different mappings evolve smoothly for ^{186}Os – ^{190}Os and undergo a pronounced reorganization at ^{192}Os . This behavior is consistent with a change of collective structure in the same mass region where independent IBM and spectroscopic studies find transitional behavior [19, 20]. Determining the precise geometric character of that change

requires the additional observables listed above.

Recent studies provide additional context for the structural evolution in the $A \approx 190$ region. Spectroscopy and lifetime measurements in ^{188}Os , interpreted with a density-functional-theory-based interacting boson-fermion model, were used to investigate the predicted prolate-to-oblate transition around $A \approx 190$ [26]. A 2024 study of neutron-rich $Z = 76$ isotopes likewise describes the osmium chain as shape transitional and finds competing prolate and triaxial structures [27]. More broadly, the mapped interacting boson model has recently been reviewed as a microscopic framework for shape transitions and coexistence in medium-heavy and heavy nuclei [28]. Beyond-mean-field calculations published in 2025 also point to enhanced prolate-oblate coexistence near $N = 116$ in rare-earth chains including Os, with quadrupole and hexadecapole collective degrees of freedom becoming strongly coupled [29]. These results strengthen the motivation for testing the present FDSM-to-boson mapping against explicit spectroscopic observables rather than inferring a unique equilibrium shape from Hamiltonian parameters alone.

7. CONCLUSIONS

We have reformulated the boson mapping of a simplified fermion dynamical symmetry model and analyzed its application to the even osmium isotopes ^{186}Os , ^{188}Os , ^{190}Os , and ^{192}Os . Three mappings were considered: the generalized Dyson mapping, a seniority-based Hermitization, and the Belyaev–Zelevinsky expansion.

The Dyson mapping provides an algebraically compact representation and preserves the relevant fermionic matrix elements in the physical boson subspace, but the mapped Hamiltonian is generally non-Hermitian in the conventional boson metric. The seniority and BZ procedures provide Hermitian IBM-like representations when the boson expansion is reduced to one- and two-body terms.

The mapped parameter sets display a common trend. Their evolution is comparatively smooth from ^{186}Os to ^{190}Os , whereas several coefficients change markedly at ^{192}Os . Because this feature appears in more than one mapping and is also reflected in the $\text{SU}(6)$ boson parametrization, it is interpreted as evidence for a rearrangement of the effective collective interaction near $A = 192$.

The present analysis does not identify the parameter rearrangement alone with a confirmed quantum phase transition. A stronger conclusion requires a systematic calculation of excitation spectra, $B(E2)$ values, quadrupole moments, and, where possible, collective energy surfaces. Such an extension would provide a direct quantitative test of how the microscopic fermion-pair structure is encoded in the effective boson description.

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