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RESONANCE STRUCTURE OF THE ^{7}Be NUCLEUS IN MICROSCOPIC TWO-CLUSTER MODEL

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Abstract. The resonant structure of the light nucleus ^{7}Be in the $^4_2$He $+^3_2$ He cluster configuration has been investigated within the framework of a microscopic two-cluster model employing the algebraic version of the resonating group method. Elastic scattering cross sections for the reaction $^7_4$Be $\rightarrow$ $^4_2$He $+$ $^3_2$He are calculated and the corresponding energy dependence is obtained. The total elastic scattering cross sections are calculated using the modified nucleon–nucleon Hasegawa–Nagata potential. The obtained resonance parameters for the available states $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ are determined and compared with experimental data. The resonance parameters for the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states are also obtained, indicating the presence of broad and narrow resonances, which indicates the existence of both short-lived and long-lived states in the nuclear system under study.

Keywords: cluster model, Resonating Group Method, resonance states, elastic scattering, nucleon–nucleon interaction, modified Hasegawa–Nagata potential.

1. Introduction

Nuclear physics, as well as nuclear astrophysics, continues to consider the study of the structure of light nuclei as one of its central and highly relevant problems. Light nuclear systems provide valuable information about the properties of nuclear forces and the mechanisms of nuclear reactions. In particular, the investigation of resonance states in light nuclei plays an important role in understanding the interaction between nuclear clusters and the dynamics of nuclear systems.

The ^{7}Be nucleus attracts considerable interest due to its cluster structure and its role in various nuclear processes. Many theoretical and experimental studies have been devoted to the investigation of its energy spectrum and resonance properties [1, 2, 3]. The cluster approach has proven to be an effective tool for describing light nuclei, where the nucleus is considered as a system of interacting clusters. Such models allow a physically transparent interpretation of the structure of nuclear states and the nature of resonances [4, 5, 6, 7, 8, 9].

In particular, the ^{7}Be nucleus can be described within the two-cluster configuration $^4_2$He $+$ $^3_2$He. This configuration is especially suitable for studying the resonance structure and scattering processes in this system. Various theoretical methods have been applied to analyze these states, including the Resonating Group Method (RGM) [10]. In the present work the resonance structure of the ^{7}Be nucleus is investigated within the microscopic two-cluster model using the algebraic version of the Resonating Group Method (AV RGM) [8, 11]. The resonance parameters for several negative-parity states are determined and compared with available experimental data [3].

2. Theoretical Model

To describe various bound and excited states of atomic nuclei in a microscopic two-cluster representation, we apply the AV RGM. The full set of generator functions is used to describe the relative motion of clusters both in connected states and in a continuum, which is a key feature of this method.

Within this approach, the wave function of a two-cluster system can be expressed in the following form [12]:

$$\Psi_J = \widehat{\mathscr{A}}\left\{[\varphi_1(A_1)\varphi_2(A_2)]_s\,\psi^J_{LS}(\vec{q})\right\} \tag{1}$$

where $\varphi_1(A_1)$ is the internal wave function of the first cluster consisting of A_1 nucleons, and $\varphi_2(A_2)$ is the internal wave function of the second cluster consisting of A_2 nucleons. Both functions depend on the spatial, spin, and isospin coordinates of individual nucleons and are antisymmetric with respect to permutations of nucleons within each cluster. In the RGM, these internal functions are assumed to be known. In this approach, the wave function of a two-cluster system is constructed using a complete set of generator functions, enabling the description of cluster relative motion in both bound and continuum states. The function $\psi^J_{LS}(\vec{q})$ is a solution of the dynamical equation and depends on the Jacobi coordinate vector $\vec{q}$, which defines the relative distance between the clusters.

Within the algebraic version of the Resonating Group Method, the wave function $\psi^J_{LS}(\vec{q})$ is expanded into an infinite series of three-dimensional harmonic oscillator functions $\psi_{nL}(q,r_0)$:

$$\psi^J_{LS}(\vec{q}) = \sum_{n=n_0}^{\infty} C_{nL}\psi_{nL}(q,r_0) \tag{2}$$

where C_{nL} are the expansion coefficients and q is the magnitude of the Jacobi vector. The explicit form of the oscillator functions $\psi_{nL}(q,r_0)$ can be found in Ref. [12]. Since the oscillator functions form an orthonormal basis, the wave function of any two-cluster system can be expanded in terms of these functions.

Taking into account (1) and (2), within the algebraic version of the RGM the total wave function of the two-cluster system can be represented as a generalized Fourier series:

$$\Psi_J = \sum_{n=n_0}^{\infty} C_{nL}\Psi_{nL} \tag{3}$$

This leads to the following system of algebraic equations for the expansion coefficients:

$$\sum_{m=n_0}^{\infty}\left[\langle\vec{n}L|\widehat{H}|\vec{m}L\rangle - E\,\delta_{n,m}\right]C_{mL} = 0 \tag{4}$$

where $\widehat{H}$ is the Hamiltonian of the nuclear system and E is the total energy. The function

$$\Psi_{nL} = \widehat{\mathscr{A}}\{\varphi_1(A_1)\varphi_2(A_2)\psi_{nL}(q,b)Y_{LM}(\vec{q})\} \tag{5}$$

represents the many-body oscillator function of the cluster system.

3. Interaction Potential

In the present work, the nucleon–nucleon interaction is described using the modified Hasegawa–Nagata potential (MHNP) [13, 14].

The MHNP provides a more detailed description of nucleon–nucleon interactions by including central, spin–orbit, and Coulomb components. The Hamiltonian of the system is written as

$$\widehat{H} = \widehat{T} + \widehat{V} \tag{6}$$

where the kinetic energy operator is defined in the center-of-mass frame.

The potential energy is represented as a sum of three contributions:

$$\widehat{V} = \sum_{j>i=1}^{A} \widehat{V}^{(\mathrm{cn})}(ij) + \sum_{j>i=1}^{A} \widehat{V}^{(\mathrm{so})}(ij) + \sum_{j>i=1}^{A} \widehat{V}^{(\mathrm{c})}(ij) \tag{7}$$

where $\widehat{V}^{(\mathrm{cn})}$ is the central interaction, $\widehat{V}^{(\mathrm{so})}$ is the spin–orbit interaction, and $\widehat{V}^{(\mathrm{c})}$ is the Coulomb interaction.

The central part of the interaction is expressed using spin–isospin projectors:

$$\widehat{V}^{(\mathrm{cn})}(ij) = \sum_{S=0,1} \sum_{T=0,1} V^{(\mathrm{cn})}_{2S+1,2T+1}(ij) \widehat{P}_S(ij) \widehat{P}_T(ij) \tag{8}$$

where S and T are the spin and isospin of the two-nucleon system.

The radial dependence of the nucleon–nucleon interaction depends on the spin and isospin channels and is represented as a superposition of Gaussian functions:

$$V^{(\nu)}_{2S+1,2T+1}(r_{ij}) = \sum_{i=1}^{N_G} V^{(\nu,i)}_{2S+1,2T+1} \exp\left\{ -\left(\frac{r_{ij}}{a^{(\nu,i)}_{2S+1,2T+1}} \right)^2 \right\} \tag{9}$$

where $V^{(\nu,i)}_{2S+1,2T+1}$, $a^{\nu,i}_{2S+1,2T+1}$ are the strength and range parameters of the interaction. The index ν specifies the type of interaction (central or spin–orbit).

4. Results and Discussions

Within the cluster approximation, the ^{7}Be nucleus is considered as a two-cluster system:

$$^7_4\mathrm{Be} = {}^4_2\mathrm{He} + {}^3_2\mathrm{He} \tag{10}$$

To perform all necessary calculations within the model, the oscillator length (b), which is the only free parameter, is determined by minimizing the energy of the two-cluster threshold. To achieve agreement with experimental data, the Majorana parameter Δm is slightly adjusted.

The optimal values of the input parameters (oscillator length b and Majorana parameter Δm) are presented in Table 1.

Table 1. Input parameters used in the calculations

Nucleus	**b, fm**	Δm	f_{LS}
$^7_4$Be	1.362	0.0002	1.000

It was also found that the spin–orbit component of the modified Hasegawa–Nagata potential is excessively strong and leads to nonphysical results. To avoid this issue, a scaling factor f_{LS} was introduced for the spin–orbit interaction and treated as a variational parameter [14]. Let us consider the elastic scattering cross sections for the reaction ${}^7_4\mathrm{Be} \rightarrow {}^4_2\mathrm{He} + {}^3_2\mathrm{He}$ for different nuclear states of the ^{7}Be nucleus.

Figure 1 shows the energy dependence of the elastic scattering cross section obtained using the MHNP.

As can be seen from Fig.1, resonance states are observed for $J^\pi = \frac{5}{2}^-$ and $J^\pi = \frac{7}{2}^-$. This indicates that the excitation of a neutron from the $p_{3/2}$ shell to the $f_{5/2}$ and $f_{7/2}$ states leads to the formation of resonance states.

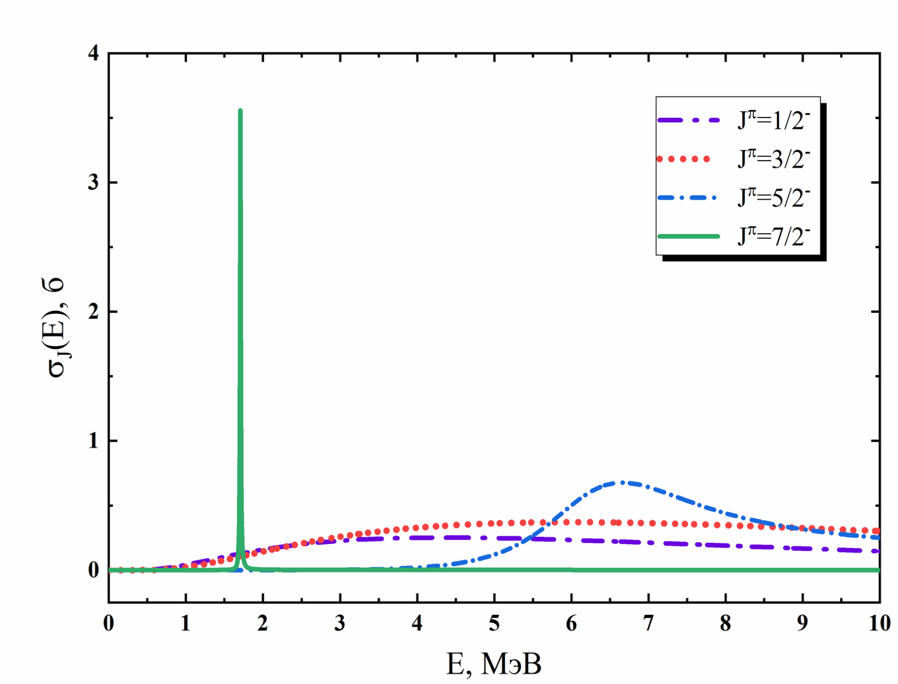

Figure 1 – Energy dependence of the elastic scattering cross section σ_J for the reaction ${}^7_4\mathrm{Be} \rightarrow {}^4_2\mathrm{He} + {}^3_2\mathrm{He}$ obtained with the MHNP.

The presence of narrow and broad resonances in the calculated cross sections reflects different physical mechanisms. Narrow resonances, such as the $\frac{7}{2}^-$ state, correspond to relatively long-lived states with small decay widths, indicating a more stable configuration of the cluster system. On the other hand, broad resonances, such as the $\frac{5}{2}^-$ state, are characterized by larger widths and shorter lifetimes, which indicates stronger coupling to the continuum. The resonances are observed within a relatively narrow energy interval; therefore, the energy range from 0 to 10 MeV was considered in the calculations. Depending on the resonance width Γ, the resonances can be classified as broad or narrow. The $J^\pi = \frac{5}{2}^-$ state corresponds to a broad resonance with a width of $\Gamma = 2.452$ MeV, while the $J^\pi = \frac{7}{2}^-$ state exhibits a sharp peak with a width of $\Gamma = 0.012$ MeV, indicating a narrow resonance.

Table 2. presents the resonance parameters of the ^{7}Be nucleus obtained using the modified Hasegawa–Nagata potential, where E denotes the resonance energy, Γ is the resonance width, and the experimental values are taken from reference [3].

Table 2. Resonance parameters of the ^{7}Be nucleus

J^π	E, MeV (MHNP)	Γ, MeV (MHNP)	E, MeV (exp.) [3]	Γ, MeV (exp.) [3]
$\frac{5}{2}^-$	6.242	2.452	5.143 ± 0.10	1.2
$\frac{7}{2}^-$	1.709	0.012	2.98 ± 50	0.175 ± 7

The results also confirm that the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states correspond to bound states of the system. As shown in Table 3, the good agreement of the calculated energies with experimental data, especially for the modified Hasegawa–Nagata potential, confirms the validity of the ${}^4_2\text{He} + {}^3_2\text{He}$ cluster description of the ^{7}Be nucleus.

Table 3. Bound states of the ^{7}Be nucleus

J^π	E, MeV (MHNP)	E, MeV (exp.) [3]
$\frac{1}{2}^-$	-0.306	—
$\frac{3}{2}^-$	-1.587	-1.588

Overall, the analysis shows that the spin–orbit interaction plays a crucial role in the correct description of resonance states in light nuclei, and its inclusion significantly improves the agreement between theoretical and experimental results.

5. Conclusion

In this work, the resonance structure of the ^{7}Be nucleus has been studied within the microscopic two-cluster model using the ${}^4_2\text{He} + {}^3_2\text{He}$ configuration. The resonance states $J^\pi = \frac{5}{2}^-$ and $J^\pi = \frac{7}{2}^-$ were identified, corresponding to broad and narrow resonances, respectively.

It was also shown that the $J^\pi = \frac{1}{2}^-$ and $J^\pi = \frac{3}{2}^-$ states are bound states, whose energies are in good agreement with experimental data. The comparison of different nucleon–nucleon potentials demonstrates that the modified Hasegawa–Nagata potential provides a more accurate description of the system.

Overall, the results confirm the validity of the cluster approach for describing the structure of the ^{7}Be nucleus.

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